GENERAL

testing statistical hypotheses worked solutions

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Introduction

In the world of data analysis, research, and evidence-based decision making, the ability to assess whether observed patterns are truly meaningful or merely the result of random chance is paramount. This is where the discipline of hypothesis testing comes into play. By framing questions in terms of null and alternative hypotheses, statisticians can apply rigorous methods to determine whether a claimed effect exists. In this comprehensive guide, we will walk through the fundamentals of hypothesis testing, present worked solutions that illustrate each step in detail, and share best practices for interpreting and communicating results. Whether you are a budding data scientist, a seasoned researcher, or simply curious about how statistical evidence is evaluated, this article will provide you with the knowledge and tools you need to master the art of testing statistical hypotheses.

Understanding Hypothesis Testing

What Is a Hypothesis?

A hypothesis is a clear, testable statement about a population parameter or the relationship between variables. It serves as the foundation for any statistical inference. Hypotheses are typically expressed in two complementary forms:

- Null hypothesis (H €) – the default assumption that there is no effect or no difference.
- Alternative hypothesis (H • or H •) – the statement that there is an effect or a difference, which the researcher seeks to support.

Null and Alternative Hypotheses

Formulating these hypotheses correctly is critical. A poorly defined null hypothesis can lead to misleading conclusions. For instance, if a pharmaceutical company claims that a new drug reduces blood pressure, the null hypothesis might state: “The mean blood pressure after treatment is equal to the mean blood pressure before treatment.” The alternative hypothesis would state: “The mean blood pressure after treatment is lower than the mean before treatment.”

Types of Tests

Depending on the nature of the data and the research question, different statistical tests are appropriate:

1. Parametric tests – assume that the data follow a specific distribution (commonly normal). Examples: t-tests, ANOVA, linear regression.
2. Non-parametric tests – make fewer assumptions about the data distribution. Examples: Mann–Whitney U test, Kruskal–Wallis test, Spearman’s rank correlation.
3. Exact tests – used when sample sizes are small, such as Fisher’s exact test.

Worked Solutions: Step by Step Examples

Below, we present a series of worked solutions that demonstrate how to carry out hypothesis tests in practice. Each example includes the research question, data description, hypothesis formulation, test selection, calculation steps, and interpretation of results. By following these detailed walk throughs, you can apply the same logic to your own

data sets.

Example 1: One Sample t Test

Scenario: A teacher claims that the average score of her students on a standardized test is higher than the national average of 75 points.

Data: Scores from 30 students: 78, 82, 77, 80, 79, … (list continues).

1. State the hypotheses

H €: ;Â Ò sR ‡F†R G ue mean equals the national average)
2. H •: ;Â fwC° 75 (the true mean is greater than the national average)

Example 2: Two Sample t Test (Independent Samples)

Scenario: A researcher wants to compare the average heights of men and women in a city.

Data: 50 men and 50 women, heights measured in centimeters.

1. State the hypotheses

H €: ;Â Ò ¼ , (no difference in average heights)
2. H •: ;Â & ¼ , (a difference exists)

Example 3: Chi Square Test for Independence

Scenario: A marketing analyst wants to determine whether customer purchase behavior is independent of age group.

Data – Contingency table of purchases vs. age categories.

1. State hypotheses

H €: Purchase behavior is independent of age group.
2. H •: Purchase behavior is dependent on age group.

Example 4: One Way ANOVA (Analysis of Variance)

Scenario: An agronomist tests whether three different fertilizers affect crop yield.

Data – Yields for each fertilizer group.

1. State hypotheses

H €: All group means are equal.
2. H •: At least one group mean differs.

Example 5: Regression Significance Test (t test for slope)

Scenario: A health economist examines whether income predicts health expenditure.

Data – Pairs of income and expenditure for 100 individuals.

1. Fit linear regression – y = ;" ² ² •x + ;Rà
2. Obtain estimated slope ;# • and its standard error SE(;# •).
3. Assume ;# • = 0.45, SE = 0.12.
4. Compute t statistic – t = ;# • / SE = 0.45 / 0.12 "H 3.75.

5. Degrees of freedom – n - 2 = 98.
6. Critical value – t €. € , …,98 "H 1.984.
7. Since 3.75 > 1.984, reject H € that ;" Ò à
8. Interpretation – Income has a significant positive effect on health expenditure.

Common Mistakes and How to Avoid Them

Even seasoned analysts can fall into pitfalls that compromise the validity of hypothesis tests. Below are frequent errors and practical solutions.

- Misinterpreting p values – A p value below 0.05 does not prove the alternative hypothesis; it merely indicates that the observed data are unlikely under the null. Always report effect sizes and confidence intervals.
- Ignoring assumptions – Parametric tests require normality, homogeneity of variance, and independence. Check residual plots, run Shapiro–Wilk tests, or use Levene's test for variance equality.
- Multiple comparisons – Conducting many tests inflates the Type I error rate. Apply Bonferroni correction, Holm–Bonferroni, or false discovery rate (FDR) adjustments.
- Small sample sizes – Small n can lead to low power. Use non parametric tests or increase sample size if feasible.
- Data dredging (p hacking) – Specifying hypotheses after seeing the data leads to over optimistic results. Pre register studies or clearly separate exploratory from confirmatory analyses.

Tools and Software for Testing Statistical Hypotheses

Modern data science offers a variety of tools that simplify hypothesis testing. Below is a brief overview of popular software packages.

- R – Comprehensive statistical environment; functions like t.test(), anova(), and chisq.test() automate many tests.

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