GENERAL

statistics probability examples and solutions

Published: September 3, 2026 | Index ID: #-

Statistics Probability Examples and Solutions: A Comprehensive Guide

Probability and statistics are the twin engines that power data driven decision making across science, business, healthcare, and everyday life. While the abstract formulas can sometimes feel intimidating, working through concrete examples and seeing step by step solutions turns theory into intuition. In this article we'll walk through the core concepts of probability, illustrate them with a variety of real world scenarios, and provide clear solutions that you can replicate. By the end you'll have a solid foundation for tackling more advanced problems, and you'll be equipped to explain statistical reasoning to colleagues, classmates, or your own curious mind.

1. Foundations of Probability

1.1 What Is Probability?

Probability is a mathematical measure that quantifies how likely an event is to occur. It is expressed as a number between 0 and 1, where 0 means impossibility and 1 means certainty. For example, the probability of flipping a fair coin and landing on heads is 0.5, or 50%.

1.2 The Sample Space and Events

The sample space, denoted by **S**, is the set of all possible outcomes of an experiment. An **event** is a subset of the sample space. For a die roll, the sample space is {1, 2, 3, 4, 5, 6}. An event could be "rolling an even number," which corresponds to {2, 4, 6}.

1.3 Basic Probability Rules

1. Complementary Rule: P(Ac) = 1 – P(A). If the probability of rolling a number greater than 4 on a die is 1/3, the probability of rolling 4 or less is 2/3.
2. Addition Rule: For mutually exclusive events A and B, P(A "* B) = P(A) + P(B). If you roll a 1 or a 6 on a die, the probability is 1/6 + 1/6 = 1/3.
3. Multiplication Rule: For independent events A and B, P(A ") B) = P(A) × P(B). The probability of flipping two heads in a row is 0.5 × 0.5 = 0.25.

1.4 Counting Principles

Counting is the backbone of probability. Two fundamental techniques are:

- Permutations: Arrangements where order matters. For example, the number of ways to seat 5 people in 5 seats is 5! = 120.
- Combinations: Selections where order does not matter. For instance, the number of ways to choose 3 students from a class of 10 is C(10,3) = 120.

These counting methods allow us to calculate probabilities in complex scenarios, such as drawing cards from a deck or selecting lottery numbers.

2. Conditional Probability and Independence

2.1 Understanding Conditional Probability

Conditional probability, denoted *P(A|B)*, measures the probability of event A occurring given that event B has occurred. The formula is:

P(A|B) = P(A ") B) / P(B)

Example: Suppose a deck of 52 cards contains 13 hearts. If you draw a heart, what's the probability that it's an Ace? There is only one Ace of hearts, so *P(Ace | Heart) = 1/13 "H 0.077*

2.2 Independence

Two events A and B are independent if the occurrence of one does not affect the probability of the other. Mathematically, *P(A ") B) = P(A) × P(B)*A classic example is rolling a die and flipping a coin; the outcome of the die does not influence the coin's result.

2.3 Bayes' Theorem

Bayes' Theorem is a powerful tool for updating probabilities based on new evidence. It states:

P(A|B) = [P(B|A) × P(A)] / P(B)

Example: In medical testing, if a disease has a prevalence of 1% (*P(Disease)*), a test has a 95% true positive rate (*P(Positive|Disease)*), and a 2% false positive rate (*P(Positive|No Disease)*), Bayes' theorem calculates the probability that a person actually has the disease given a positive test result.

3. Random Variables and Distributions

3.1 Discrete Random Variables

A discrete random variable takes on a countable set of values. Common distributions include:

- Bernoulli Distribution: Models a single binary experiment (e.g., success/failure).
- Binomial Distribution: Counts the number of successes in n independent Bernoulli trials. For example, the number of heads in 10 coin flips.
- Geometric Distribution: Models the number of trials until the first success.
- Poisson Distribution: Describes the number of events in a fixed interval when events occur independently at a constant rate.

3.2 Continuous Random Variables

Continuous random variables can take on any value within an interval. Key distributions include:

- Uniform Distribution: All outcomes in an interval are equally likely.
- Normal (Gaussian) Distribution: The "bell curve" shape; many natural phenomena approximate this distribution.
- Exponential Distribution: Models time between independent events occurring at a constant rate.

3.3 Expected Value and Variance

The **expected value (mean)** of a random variable X, denoted *E[X]*, represents the long run average outcome. For a discrete variable:

E[X] = :2 ·, r …, Ò ,•Ð

The **variance** measures dispersion around the mean:

Var(X) = E[(X – E[X])²] = :2 ²‡, b Uµ…Ò›" r …, Ò ,•Ð

Standard deviation is the square root of variance.

4. Probability Distributions in Action

4.1 The Binomial Example: Rolling Dice

Suppose you roll a fair six sided die 12 times. What's the probability of rolling exactly four 6's?

Solution:

1. Each roll is a Bernoulli trial with success probability p = 1/6.
2. The number of successes follows a binomial distribution: X ~ Binomial(n = 12, p = 1/6).
3. The probability mass function is P(X = k) = C(n, k) p^k (1-p)^{n-k}.
4. Plugging in k = 4: P(X = 4) = C(12, 4) (1/6)^4 (5/6)^8 "H 0.200.

4.2 Poisson Example: Customer Arrivals

A coffee shop receives an average of 4 customers per hour. What's the probability that exactly 7 customers will arrive in the next hour?

Solution:

1. Let ;² Ò B † verage rate).
2. The Poisson PMF: P(X = k) = (e^{-;·Ò »^k) / k!.
3. For k = 7: P(X = 7) = (e^{-4} 4^7) / 7! "H 0.072.

4.3 Normal Approximation: Test Scores

Test scores for a large class are normally distributed with mean 75 and standard deviation 10. What proportion of students score above 85?

Solution:

1. Standardize the score: Z = (X – ;Â' ò Ã = (85 – 75) / 10 = 1.0.
2. Use the standard normal table: P(Z > 1.0) "H 0.1587.
3. Thus, about 15.9% of students score above 85.

5. Real World Applications

5.1 Healthcare Diagnostics

Probability helps clinicians assess disease risk, interpret test results, and make treatment decisions. For instance, evaluating the likelihood of a disease given a positive test result relies on Bayes' theorem and prevalence data.

5.2 Finance and Risk Management

Statistical models estimate portfolio volatility, credit risk, and market trends. The normal distribution underpins many risk metrics, while the binomial model informs option pricing.

5.3 Quality Control in Manufacturing

Control charts use probability to detect deviations from process stability. The Poisson distribution models defect counts, and the binomial distribution assesses the probability of a certain number of defective items in a batch.

5.4 Sports Analytics

Probability informs player performance predictions, game outcome forecasts, and betting odds. For example, the Poisson model predicts the number of goals a football team might score in a match.

6. Step by Step Problem Solving

6.1 Problem: Lottery Odds

You buy a ticket in a 6/49 lottery (choose 6 numbers out of 49). What's the probability of winning the jackpot (matching all 6 numbers)?

Solution:

1. The total number of distinct 6 number combinations is C(49, 6) = 13,983,816.
2. Only one of those combinations is the winning set.

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